

Detection of space-time fluctuations by a model matter interferometer

by

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Abstract

In papers on primary state diffusion (Percival 1994, 1995), numerical estimates suggested that fluctuations in the space-time metric on the scale of the Planck time ($\approx 10^{-44}$ s) could be detected using atom interferometers. In this paper we first specify a stochastic metric obtained from fluctuations that propagate with the velocity of light, and then develop the non-Markovian quantum state diffusion theory required to estimate the resulting decoherence effects on a model matter interferometer. Both commuting and non-commuting fluctuations are considered. The effects of the latter are so large that if they applied to some real atom interferometry experiments they would have suppressed the observed interference. The model is too crude to conclude that such fluctuations do not exist, but it does demonstrate that the small numerical value of the Planck time does not alone prevent experimental access to Planck-scale phenomena in the laboratory.

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1. Introduction

Two earlier papers on primary state diffusion, PSD1 (Percival 1994), and PSD2 (1995) describe an alternative quantum theory, and a brief guide to the literature in the field. In PSD1 the theory had one free parameter, a time τ_0 , which in PSD2 is close to the Planck time. Thus PSD2 has no free parameters. This paper may be considered as a sequel to PSD1 and PSD2, in which the proposal to use atom interferometry to test the validity of the theory is worked out in greater detail, with both commuting and non-commuting fluctuations. There is a large literature on experimental constraints on alternative quantum theories, but most are concerned with estimates or modifications of free parameters (e.g. Ghirardi, Rimini and Weber 1986, Pearle and Squires 1994, 1995). Exceptions are the theories of (Diósi 1987, 1992) and (Penrose 1986, 1996), and of Ellis and his collaborators discussed below.

We also have a purpose quite separate from any alternative quantum theories, to demonstrate that atom interferometers might be used to put an experimental bound on Planck scale space-time fluctuations (Ellis *et al.* 1984).

It was principally Einstein's 1905 theory of Brownian motion and the subsequent experiments of Perrin and others that established without question the reality of atoms (Einstein 1956, Pais 1965). Brownian motion is a diffusion process, and because of this, measurements on a macroscopic scale could be used to determine quantities on an atomic scale. So by analogy with Brownian motion, we should look for a diffusion process which enables us to determine quantities on the Planck scale by experiments on the atomic scale. Space-time fluctuations produce a diffusion in quantum amplitudes (PSD2). We use a relativistic theory of non-commuting space-time fluctuations in two dimensions and its detailed application to a simple model of an atom interferometer, to show that the small numerical value of the

Planck time does not alone prevent experimental access to Planck scale phenomena in the laboratory. Section 2 discusses the relation between Brownian motion and space-time fluctuations.

For finite intervals that are much longer than τ_0 , the space-time fluctuations are dominated by drift, and the space-time will appear to be nearly smooth. Nevertheless Planck scale fluctuations contribute to the action integrals in the exponents of all quantum path summations. In principle, and possibly in practice, these could be detected by atom interferometry. If we could detect the fluctuations we could measure the value of τ_0 , which would provide valuable information about dynamics on Planck scales.

In order to do this we treat simple examples of linear non-Markovian quantum state diffusion theory. We treat three cases: commuting delta-fluctuations, commuting propagating fluctuations and non-commuting propagating fluctuations. Delta-fluctuations are introduced by Ghirardi, Grassi and Pearle (1990), whereas non-Markovian fluctuations and (different) operator fluctuations are discussed by Pearle (1993).

Quantum state diffusion, or QSD, represents an open quantum system by a pure state diffusing in Hilbert space, as a practical alternative to the representation in terms of a density operator (Gisin and Percival, 1992 1993a,b, Percival 1994). Our formal theory and applications are based on the linear theory of quantum state diffusion which was developed independently of the nonlinear theory and for a different purpose (Barchielli and Belavkin 1991, Goetsch and Graham 1994). The nonlinear theory has the advantages that it is realistic in the sense that a physical system is represented directly by a pure quantum state. It can be applied to single runs of a laboratory experiment, and is excellent for computations (Schack, Brun and Percival, 1995). The linear theory does not have these advantages, but is simpler analytically (Strunz 1996), which is why we use it here. Section 3 introduces linear Markovian QSD with complex fluctuations in the notation of the usual nonlinear theory of Gisin and Percival, and then introduces QSD with non-Markovian simple repeated and persistent fluctuations.

Special relativity was essential to quantum state diffusion as a fundamental theory (Gisin 1984), yet it has proved difficult to develop a satisfactory relativistic theory. Nonrelativistic primary state diffusion (PSD1, PSD2) is one of the realistic quantum theories, in which classical systems, quantum systems and the quantum measurement process are all represented by state vectors that satisfy the Schrödinger equation to a good approximation on atomic scales. They localize on a macroscopic scale through a universal diffusion process, with the same mathematical structure as nonlinear QSD. In these earlier nonrelativistic formulations this universal diffusion is the result of fluctuations which are functions of time only, with a conjecture as

to the consequences of relativity. Here we provide simple examples of relativistic fluctuations. Any relativistic generalization of a realistic theory like PSD presents both conceptual and technical problems (Bell 1990, Diósi 1990, Ghirardi, Grassi & Pearle 1990). We do not treat the conceptual problems here, but solve some of the technical problems that arise because in special relativity the fluctuations depend on both space and time.

According to nonrelativistic primary state diffusion, and by analogy with Brownian motion, there is no correlation between the fluctuations at different times. Relativistically, to preserve Lorentz covariance, this means that there is no correlation over any timelike interval. There is no such direct constraint for spacelike intervals, but nevertheless we shall assume that the only correlations are for null intervals. We principally treat the case in which the fluctuations propagate like a scalar field without interaction, so that there is correlation for all null intervals. For the special case of 2D space-time the correlation between any pair of different points on a light line is then the same. At the other extreme is the case of delta-correlations in space-time (Ghirardi, Grassi and Pearle 1990), where the fluctuations do not propagate at all, and which is discussed in the final section.

Among the theories that base a universal decoherence mechanism in quantum mechanics on gravitation is the work of Diósi (1987, 1989 and 1992). He relates fundamental quantum uncertainties in the Newtonian gravitational potential of quantum systems (and thus in the time-time component of the metric) to the suppression of coherence in their (macroscopic) quantum superpositions. The relevant decoherence rate is given by the gravitational interaction energy of the superposed states. Similar ideas are developed by Penrose (1986, 1996), who relates gravitation and wave function collapse.

The research of Ellis and his collaborators is mainly based on Planck scale physics and does not depend on any free parameters. They have suggested that tests of quantum gravity can be made using the $K\bar{K}$ system, and experiments have now been made (Ellis, Hagelin, Nanopoulos, & Srednicki 1984, Adler et al, 1995). They have also suggested that coherence in SQUIDS should be used (Ellis, Mohanty & Nanopoulos 1990). According to PSD the energy difference in both cases is far too small for the decoherence due to Planck scale space-time fluctuations to be detected, so such experiments might help to distinguish the two theories. Their 1984 paper also suggested that neutron interferometry should be used, and the experiments described here are in part a sequel to that suggestion. The 1990 paper emphasized the absence of a reliable computational scheme for quantum gravity. This is consistent with our approach, which starts from the relatively simple conditions that must be satisfied by realistic quantum theories with no free parameters. It seems that the problem of uniting quantum gravity with quantum foundations can be attacked

from both sides (Amelino-Camelia, Ellis, J. & Mavromatos 1996).

Section 4 presents a general formal theory with a Dyson expansion for the state vectors of the ensemble, which leads to an expansion for the density operator. The application of this general theory to particular cases depends strongly on the nature of the problem. Section 5 shows how the selfadjoint part of the drift operator is determined by the trace condition on the density operator, and the following section 6 contains expressions for means over products of elementary fluctuations, in which the first effects of noncommuting fluctuations are seen.

Section 7 treats in detail a crude model of a matter interferometer with one space dimension. For this model propagating commuting fluctuations produce no effect, due to cancelations between the two arms of the interferometer, whereas non-commuting fluctuations and delta-fluctuations lead to a suppression of interference. The special nature of this model leads to some helpful simplifications. Section 8 applies the theory of the model to the experiment of Kasevich and Chu (1992), and concludes that it puts severe constraints on the magnitude and nature of the space-time fluctuations, subject to the assumptions made.

2. Brownian motion and space-time fluctuations

The vertical displacement of a typical Brownian particle is made up of two parts: fluctuations and drift. The fluctuations are due to collisions with molecules. These are processes on the atomic scale, producing random upward and downward displacements. The drift, due to the force of the Earth's gravitational field, is uniformly downwards. In a time interval Δt , the mean square displacement $M|\Delta s|^2$ due to the diffusion is proportional to Δt , so a typical displacement is proportional to the square root of the time. In the same time interval the drift is proportional to the time itself. Consequently for sufficiently short time intervals the motion is dominated by diffusion, and for longer intervals it is dominated by the drift. There is a characteristic time interval τ that marks the approximate boundary between the smaller time intervals Δt for which the diffusion dominates and the larger time intervals for which the drift dominates. The measured value of τ provided valuable information about atoms. For times T much longer than τ , the fluctuations are typically about $(\tau/T)^{1/2}$ of the displacement due to the drift.

Space-time is smooth and curved on large scales. The change in the motion of particles as a result of the curvature of space-time is seen as the force of gravity. The curvature also produces changes in the proper time measured by clocks. But space-time cannot be smooth on Planck scales, because of quantum fluctuations.

For Brownian motion, Einstein was able to obtain relatively simple relations without detailed dynamics on the atomic scale, which no-one in his time understood sufficiently. The corresponding relations for space-time fluctuations depend on the

detailed dynamics of space-time on Planck scales, like the method of quantization, and the topology of space-time foam (Hawking 1982, Ellis et al 1984). Ellis and his collaborators have based their approach to alternative quantum theories on this dynamics. Here, following Einstein, we try to obtain relatively simple relations without the detailed dynamics.

We treat space-time classically, but assume that quantum effects produce stochastic fluctuations on the scale of the Planck time. Proper time is then made up of two parts: fluctuations and drift. The fluctuations are due to Planck scale quantum processes, producing random changes in the proper time. This is added to the drift, which is just the proper time when space-time is flat. Consequently for sufficiently short time intervals near to the Planck time the proper time fluctuates strongly. For longer intervals it is dominated by the drift and looks smooth. There is a time τ_0 which marks the approximate boundary between the smaller time intervals Δt for which the diffusion dominates and the larger time intervals for which the drift dominates. It is generally assumed that the space-time fluctuations are connected with quantum gravity, so we would expect τ_0 to be within a few orders of magnitude of the Planck time, about $5 \times 10^{-44}\text{s}$.

For significantly larger scales, a proper time interval Δs for a timelike segment of space-time is represented to a good approximation by

$$\Delta s = \Delta \bar{s} + \tau_0^{1/2} \Delta \xi(x), \quad (2.1)$$

where $|\Delta \xi(x)|^2 = \Delta \bar{s}$. The barred proper time is for the Minkowski metric, and the fluctuations $\Delta \xi(x)$ depend on the space-time point x . Notice that this is not the same as in PSD2, which considers only the time-time components of the metric. The formulation here corresponds to the multiplication of a smooth metric by an external fluctuating factor. Thus the fluctuations produce a conformal change in an otherwise smooth (e.g. Penrose and Rindler, 1984), as proposed by Sánchez-Gómez (1994). This leaves null intervals unchanged, and so has the consequence that fields like light in a vacuum, corresponding to particles of zero rest-mass, are unaffected.

There are many theories of space-time with non-commuting metrics (Connes 1995) or an equivalent, including superstring theory (Green, Schwarz and Witten, 1987), and supersymmetric GUT (Lopez, Nanopoulos and Zichichi 1994). These are inspired by the need to unite particle theory with gravity. The metrics chosen for this paper were not based on any of these, but were obtained by criteria of simplicity and accessibility by measurements, from the ideas of alternative quantum theories in general and primary state diffusion theory in particular. The non-commuting operators are in an isospace, which is attached to the space-time and not to the matter in it. As a consequence the factor in the isospace for the density operators of matter is the trivial unit operator, which remains unchanged by the dynamics.

3. Linear Markovian and non-Markovian QSD

Linear QSD is obtained from the nonlinear theory by omitting the nonlinear terms. In this theory, there is no direct correspondence between the quantum states and the states of physical systems, so it is not a realistic theory in the sense used by Einstein Podolsky and Rosen (1935) and by Bell (1987). The states are not normalized, and each state carries a weight which is given by its norm. However the density operator is still given by the same expression as for nonlinear QSD, that is

$$\rho(t) = M|\psi(t)\rangle\langle\psi(t)|, \quad (3.1)$$

where M represents a mean over the ensemble. This is discussed by Ghirardi, Grassi and Pearle (1990). It is important to recognize that the normalized states of nonlinear QSD are *not* given by normalizing the states of linear QSD, but that corresponding to every linear QSD theory there is a nonlinear QSD theory which gives the same density operator and the same master equation.

Here we introduce the linear theory in the notation of the earlier Markovian non-linear quantum state diffusion theory, without the complications that appear later in this paper. First consider the usual case in which each fluctuation is a complex scalar $d\xi_j(t)$ which is applied only once, between the times t and $t + dt$, with the ensemble mean orthonormality properties

$$\begin{aligned} M d\xi_j(t) &= M d\xi_j(t)^2 = 0 \\ M d\xi_j(r) d\xi_k^*(t) &= \delta_{jk} \delta_{rt} dt \end{aligned} \quad (3.2)$$

where by the conventions of the Itô calculus, this is to the lowest order in dt .

The linear QSD equations are then obtained from the nonlinear equations in Gisin and Percival (1993a) by removing the nonlinear parts:

$$|d\psi\rangle = \left[- (i/\hbar) H(t) dt - \frac{1}{2} \sum_j L_j^\dagger(t) L_j(t) dt + \sum_j L_j(t) d\xi_j(t) \right] |\psi\rangle, \quad (3.3)$$

where the $\Delta\xi_j(t)$ are Itô stochastic differentials. From now on all the state diffusion equations are linear QSD equations.

Unlike nonlinear QSD, the linear equations do not preserve the norm of the state vector, but nevertheless the density operator obtained from the weighted mean (3.1) still satisfies the master equation

$$\begin{aligned} d\rho(t)/dt &= -(i/\hbar)[H(t), \rho(t)] \\ &+ \sum_j \left[- \frac{1}{2} L_j^\dagger(t) L_j(t) \rho(t) - \frac{1}{2} \rho(t) L_j^\dagger(t) L_j(t) + L_j(t) \rho(t) L_j^\dagger(t) \right]. \end{aligned} \quad (3.4)$$

The general form of linear quantum state diffusion equation is

$$|d\psi\rangle = dG(t)|\psi(t)\rangle = dR(t)|\psi(t)\rangle + dF(t)|\psi(t)\rangle, \quad (3.5)$$

where $dG(t)$ is the linear differential evolution operator, made up of a drift part $dR(t)$ and a fluctuation part $dF(t)$ whose ensemble mean is zero:

$$MdF(t) = 0. \quad (3.6)$$

The fluctuation operator $dF(t)$ we assume to be given by the physics of the problem. For Markovian QSD it is given by the last sum of equation (3.3). The drift operator $R(t)$ is differentiable so $dR(t) = \dot{R}(t)dt$, and $\dot{R}(t)$ can be divided into its self-adjoint and skew-adjoint parts, the latter being given by the Hamiltonian:

$$\begin{aligned} \dot{R}(t) &= \dot{R}_R(t) + i\dot{R}_I(t) \\ \dot{R}_I(t) &= -H(t)/\hbar \end{aligned} \quad (3.7)$$

The self-adjoint part of the drift operator is obtained from the so-called trace condition, that the trace of the density operator ρ must be conserved for arbitrary ρ . For Markovian QSD this gives

$$\begin{aligned} 0 &= \text{Tr } d\rho \\ &= \text{Tr} [|\psi\rangle\langle d\psi| + |d\psi\rangle\langle\psi| + |d\psi\rangle\langle d\psi|] \\ &= \text{Tr } \rho \left[\dot{R} + \dot{R}^\dagger + \sum_j L_j^\dagger L_j \right] dt \\ \dot{R}_R &= -\frac{1}{2} \sum_j L_j^\dagger L_j, \end{aligned} \quad (3.8)$$

from which the linear Markovian QSD equation (3.3) follows. In this way the trace condition is used to obtain both the self-adjoint part of the drift term and the resultant master equation. Sections 5 and 7 use the trace condition in the same way for non-Markovian noncommuting fluctuations.

For linear Markovian QSD the differential fluctuation operator $dF(t)$, has the form

$$dF(t) = \sum_j L_j(t) d\xi_j(t). \quad (3.9)$$

$F(t)$ is not differentiable, despite the Itô notation that suggests that it is.

From the stochastic relations (3.2) we obtain for the differential fluctuation operators themselves:

$$\begin{aligned} \text{M}dF(t) = 0, \quad \text{M}(dF(r)dF(t)) = 0, \quad \text{M}(dF(r)dF^\dagger(t)) = X(r)\delta_{rt}dt, \\ \text{(Markov)} \end{aligned} \quad (3.10)$$

where $X(t)$ is a time-dependent operator. Equations (3.10) can be taken as the definition of Markovian QSD and are particularly useful because they are valid when the fluctuations $d\xi$ are themselves noncommuting operators, and they can be modified to define non-Markovian QSD with repeated or persistent fluctuations.

Now we remove the Markovian condition, so that the fluctuations can repeat or persist over a period of time. We are interested in applications to laboratory experiments, for which the effects of the fluctuation are small, and so a Dyson expansion can be used for the fluctuating quantum states. The measured effects appear in the density operator, which is derived by taking the ensemble mean over the perturbed states.

If any fluctuation affects the system at more than one time, then there is correlation between the fluctuations at different times, and the diffusion is non-Markovian. In the simplest case, which is applicable to the matter interferometer model of this paper, each fluctuation appears just twice. This is the example of simple repeated fluctuations

The stochastic relations are then

$$\begin{aligned} \text{M}dF(t) = 0, \quad \text{M}(dF(t)dF(r)) = 0, \\ \text{M}(dF(t)dF^\dagger(r)) = (X^0(r)\delta_{tr} + X^+(r)\delta_{t^+(t),r}X^-(r)\delta_{t^-(t),r})dt, \\ \text{(simple repeated)} \end{aligned} \quad (3.11)$$

where a fluctuation which affects the system at time t also affects it at the later time $t^+(t)$ and the earlier time $t^-(t)$, and at no other times. These repeated fluctuations are needed for the two-dimensional matter interferometer model of Section 7.

The theory of simple repeated fluctuations can be generalized to include several repetitions and continuously variable time delays.

4. Dyson expansion

By standard formal Dyson perturbation methods, if K is the propagator for the state vector, so that

$$|\psi(t)\rangle = K(t,0)|\psi(0)\rangle, \quad (4.1)$$

then

$$\begin{aligned}
K(t, 0) &= \mathcal{T} \exp \left(\int_0^t dG(r) \right) \\
&= \mathcal{T} \exp G(t) \\
&= \sum_n K^{(n)}(t, 0),
\end{aligned} \tag{4.2}$$

where \mathcal{T} is the time-ordering operator.

We assume that for time $t = 0$

$$\xi(0) = 0 \tag{4.3}$$

for all fluctuations ξ , and that

$$R(0) = 0. \tag{4.4}$$

These are conventions which simplify the analysis.

In the second line of (4.2) the operator $G(t)$ in the exponent represents the integral

$$G(t) = \int_{r=0}^t dG(r) \tag{4.5}$$

and the time ordering operator \mathcal{T} refers to the ordering of the operators $dG(r)$.

When all operators commute, the integrals in the exponent can be reordered in any way, and in particular all the operators corresponding to each independent fluctuation can be associated with a single time. So for this model the repeated QSD equations then become equivalent to ordinary Markovian QSD. A trivial but useful example is where $G(t) = cI$, for all t , in which case the density operator remains unchanged. This occurs for the matter interferometry model of Section 7 with commuting fluctuations.

On expanding the exponential in (4.2), we get

$$\begin{aligned}
K(t, 0) &= \mathcal{T} \left(I + \int_{r=0}^t dG(r) + \frac{1}{2!} \int_{r=0}^t dG(r) \int_{r=0}^t dG(s) + \dots \right) \\
&= \left(I + \int_{r=0}^t dG(r) + \int_{r=0}^t dG(r) \int_{s=0}^r dG(s) + \dots \right) \\
&= \left(I + \int_{r=0}^t dG(r) + \int_{r=0}^t dG(r) \cdot G(r) + \dots \right).
\end{aligned} \tag{4.6}$$

The initial conditions and iteration for $K^{(n)}(t, 0)$ are given by

$$K^{(0)}(t, 0) = I, \quad K^{(1)}(t, 0) = G(t), \quad K^{(n)}(t, 0) = \int_0^t dG(r) K^{(n-1)}(r, 0), \tag{4.8}$$

where the limits of integrals are all labeled by the time, so for $K^{(n)} = K^{(n)}(t, 0)$, we have

$$K^{(0)} = I, \quad K^{(1)} = G(t), \quad K^{(2)} = \int_0^t dG(r).G(r). \quad (4.9)$$

Despite appearances, $K^{(2)}$ cannot usually be integrated to give $G(t)^2/2$, because $dG(r)$ does not usually commute with $G(r)$.

The first few terms in the expansion of the density operator ρ_t at time t are

$$\begin{aligned} \rho_t &= MK(t, 0)\rho_0 K(0, t) \\ &= M(\rho_0 \\ &\quad + K^{(1)}\rho_0 + \rho_0 K^{(1)\dagger} + K^{(1)}\rho_0 K^{(1)\dagger} \\ &\quad + K^{(2)}\rho_0 + \rho_0 K^{(2)\dagger} + K^{(2)}\rho_0 K^{(1)\dagger} \\ &\quad \quad + K^{(1)}\rho_0 K^{(2)\dagger} + K^{(2)}\rho_0 K^{(2)\dagger} \\ &\quad + \dots), \quad (K^{(n)} = K^{(n)}(t, 0)). \end{aligned} \quad (4.10)$$

Section 6 shows that the fluctuations contribute to the diagonal terms $K^{(n)}\rho_0 K^{(n)\dagger}$ only.

5. Trace condition

The operator G has to be expressed in terms of the drift and fluctuation operators R and F , but the drift terms are not known until the trace condition is applied. The only terms in the expansion that can be obtained directly are the fluctuation and Hamiltonian terms. The self-adjoint part of the drift operator has to be evaluated simultaneously with the term by term evaluation of the density operator, which makes things complicated.

However it often happens that the only significant term is the first non-zero term. This is certainly true for the applications that we have in mind, for which *any* nonzero effect of the fluctuations would be important. In that case the procedure is relatively simple.

The propagator with the drift terms set to zero is the pure fluctuation propagator, denoted $K_F(t, 0)$, for which the entire above theory applies with G replaced by F . Let ρ_{Ft} denote the density operator obtained from the fluctuation propagator alone,

$$\rho_{Ft} = MK_F(t, 0)\rho_0 K_F^\dagger(t, 0). \quad (5.1)$$

The expansion of this partial density operator can be simplified, because the off-diagonal terms $MK_F^{(n)}\rho_0 K_F^{(n')\dagger}$ are zero for different n and n' . This follows from

the theory of unbalanced means presented in the next Section. The expansion then becomes

$$\rho_{Ft} = \sum_n \mathbb{M} K_F^{(n)} \rho_0 K_F^{(n)\dagger}. \quad (5.2)$$

This is expanded up through the first non-zero term, labeled by $n = n_1$, which is then used to obtain the corresponding drift to the same order, using the condition that the trace of ρ is 1 for arbitrary initial ρ_0 , giving

$$0 = \text{Tr}(\rho_t - \rho_0) \approx \text{Tr}(S\rho_0 + \rho_0 S^\dagger + \mathbb{M} K_F^{(n_1)} \rho_0 K_F^{(n_1)\dagger}), \quad (5.3)$$

so that the self-adjoint part of S is

$$S_R \approx -\frac{1}{2} K_F^{(n_1)\dagger} K_F^{(n_1)}. \quad (5.4)$$

It follows that any term in the expansion of ρ_{Ft} which is proportional to ρ_0 has no effect on ρ_t because it is canceled by the corresponding drift term which comes from the trace condition. This helps to simplify the analysis.

6. Commuting and non-commuting fluctuations

In this section we obtain means over products of commuting and noncommuting fluctuations. Here we use differences instead of differentials, and then take the small time limit, because the limiting processes are subtle. Before taking the limit the equalities are correct only to leading order in powers of Δt . The discrete times t are separated by multiples of the interval Δt . We later use the limit ‘lim’ which is always to be understood in the sense of ‘ $\lim_{\Delta t \rightarrow 0}$ ’.

In earlier versions of QSD each fluctuation $\Delta\xi$ was supposed to be a complex scalar with distribution invariant under a complex rotation around the origin, represented by a multiplying factor of modulus unity. A point in the space of such fluctuations can be represented by one complex parameter. This is generalized to non-commuting fluctuations which have the corresponding statistical properties with the complex conjugate replaced by an adjoint. The number of complex parameters is N and they can be chosen so that the distribution of the fluctuations is invariant under rotation about the origin in a complex Euclidean parameter space, an iso-space independent of space-time. It is helpful to think of the distribution as a Gaussian in this parameter space. The means over products, which appear in the density operator, always reduce to a real number times the unit operator in the iso-space. The operators X of Section 3 have the same trivial factor.

If j labels a set of independent fluctuations, then they satisfy the basic stochastic equalities

$$\begin{aligned} \text{M}\Delta\xi_j(t) &= 0 & (\text{a}) \\ \text{M}\Delta\xi_j(r)\Delta\xi_k(t) &= 0 & (\text{b}) \\ \text{M}\Delta\xi_j(r)\Delta\xi_k^\dagger(t) &= \delta_{jk}\delta_{rt}\Delta t & (\text{c}), \end{aligned} \tag{6.1}$$

which should be compared with equation (3.2) for commuting fluctuations. Notice that (a) and (b) follow from rotational invariance, since almost every combined rotation in the parameter spaces of the fluctuations changes the value of the mean unless that value is 0. The same goes for the zero off-diagonal elements in (c), for which the diagonal element provides a normalization condition.

In the same way the rotational invariance shows that an unbalanced mean is zero:

$$\text{M}\Delta\xi_j(t)^n\Delta\xi_j^\dagger(t)^m = 0 \quad (n, m \text{ different}). \tag{6.2}$$

This result is used in eqn. (5.2) to simplify the expansion of $K_F(t)$ by removing the off-diagonal terms with different n, n' from the expansion (4.10).

The second order expansion of K_F leads to products of four fluctuations for ρ_F . This requires the mean

$$\text{M}\Delta\xi(r')\Delta\xi(r)\Delta\xi^\dagger(t)\Delta\xi^\dagger(t'), \tag{6.3}$$

which is zero unless

$$\begin{aligned} & r = t \quad \text{and} \quad r' = t' & (\text{direct}) \\ \text{or} & \quad r = t' \quad \text{and} \quad r' = t & (\text{exchange}). \end{aligned} \tag{6.4}$$

These are called the direct and exchange terms as shown. The direct term is independent of the commutation properties of the fluctuations. It can be evaluated directly to give Δt^2 by first taking the mean over the fluctuations $\Delta\xi(t)$ and then over $\Delta\xi(t')$. The exchange term depends on the commutation properties of the fluctuations. If they commute, then it is the same as the direct term. Otherwise we deal with each case separately.

In this paper we restrict the detailed theory to the special case of $N = 3$ where the fluctuations are derived from Pauli matrices σ_i . These will be called Pauli fluctuations. Then in order to satisfy the basic relations (6.1) the $\Delta\xi(t)$ are given by

$$\Delta\xi(t) = \frac{1}{\sqrt{3}} \sum_i \Delta\xi_i(t)\sigma_i, \tag{6.5}$$

$$\text{where} \quad \Delta\xi_i(r)\Delta\xi_j^\dagger(t) = \delta_{ij}\delta_{rt}\Delta t$$

so that the 3 complex components $\Delta\xi_i(t)$ are statistically independent fluctuations. They can be considered as components of a complex 3-vector fluctuation in an iso-space.

The first product in the Dyson expansion that depends on the commutation properties is the second order exchange term, which is

$$\begin{aligned}
& M\Delta\xi(t')\Delta\xi(t)\Delta\xi^\dagger(t')\Delta\xi^\dagger(t) \\
&= \frac{1}{9}M \sum \Delta\xi_{i'}(t')\sigma_{i'}\Delta\xi_i(t)\sigma_i\Delta\xi_{j'}^\dagger(t')\sigma_{j'}\Delta\xi_j^\dagger(t)\sigma_j(\Delta t)^2 \\
&= \frac{1}{9} \sum \delta_{i'j'}\delta_{ij}\sigma_{i'}\sigma_i\sigma_{j'}\sigma_j(\Delta t)^2 \\
&= \frac{1}{9} \sum \sigma_{i'}\sigma_i\sigma_{i'}\sigma_i(\Delta t)^2 \\
&= \frac{1}{9}(-3)(\Delta t)^2 = -\frac{1}{3}(\Delta t)^2,
\end{aligned} \tag{6.6}$$

where sums are over all suffixes and arguments. In the last sum there are 3 products in which $i' = i$ and $\sigma_i^4 = 1$, and 6 products in which i and i' are not equal, with the value $(\sigma_1\sigma_2)^2 = -\sigma_3^2 = -1$.

For arbitrary fluctuations that satisfy the conditions of this section,

$$M\Delta\xi(t')\Delta\xi(t)\Delta\xi^\dagger(t')\Delta\xi^\dagger(t) = \eta(\Delta t)^2, \tag{6.7}$$

for some constant η which depends on the commutation properties of the fluctuations. For commuting fluctuations and for the Pauli fluctuations above we have

$$\eta(\text{commuting}) = 1, \quad \eta(\text{Pauli}) = -1/3. \tag{6.8}$$

We do not consider cases with all times identical, for example

$$M\Delta\xi(t)\Delta\xi(t)\Delta\xi^\dagger(t)\Delta\xi^\dagger(t), \tag{6.9}$$

because although they are of the same order as the direct and exchange terms, their contribution tends to zero with Δt .

Summarizing for the second order:

$$M\Delta\xi(r')\Delta\xi(r)\Delta\xi^\dagger(t)\Delta\xi^\dagger(t') = (\delta_{r't'}\delta_{rt} + \eta\delta_{rt'}\delta_{r't})(\Delta t)^2. \tag{6.10}$$

7. Interferometer model

Here we consider a specific model which is a crude representation of a matter interferometer in a fluctuating field. The wave packet in each arm of the interferometer is represented by a single state, with label 1 on the left and 2 on the right. These states have orthogonal projection operators P_1 and P_2 . For the purposes of this paper the size of the wave packets is supposed sufficiently small that they can be considered to be at points in space. The theory is worked out for 1 space dimension only. The remaining dimensions complicate the theory, but they do not affect the orders of magnitude of the results, which is all that we are concerned with here. For the atom interferometers of interest, the velocities are typically of order 1ms^{-1} , so we neglect the effects of time dilation due to these velocities here.

Let the paths of the wave packets be $x_j(t)$, $j = 1, 2$. Choose the origin of time when the wave packets separate, and let T be the drift time before they recombine to produce an interference pattern, so that

$$x_1(0) = x_2(0), \quad x_1(T) = x_2(T) = 0. \quad (7.1)$$

An interaction representation is used in which the basis for each wave packet is the unperturbed wave packet, so that the interaction Hamiltonian is zero.

We use time units to measure distances, with the velocity of light $c = 1$, so that $1\text{ns} \approx 0.3\text{m}$. For an atom interferometer whose wave packet separations are produced by photon recoil, wave-packet separations might be about 0.1ns , whereas the drift time T is of the order of 1s .

The delta-fluctuations for the two wave packets are independent of one another, so the QSD equation and the master equations are (3.3) and (3.4) with $H = 0$, $L_1 = \Gamma^{1/2}P_1$, $L_2 = \Gamma^{1/2}P_2$. As a result, the off-diagonal elements of ρ decay exponentially with decay constant Γ , where Γ is the inverse of the decoherence time defined in PSD1,

$$\Gamma = T_p^{-1} = (mc^2)^2 \tau_0 \hbar^{-2}. \quad (7.2)$$

Thus, for delta-fluctuations, the interference pattern is suppressed significantly if the drift time T of the wave packets is not less than Γ^{-1} .

In the case of the commuting propagating fluctuations, for each time t there are two impulsive fluctuations $\Delta\xi^+(t)$ and $\Delta\xi^-(t)$ that reach wave packet 1 at time t , which last for a time Δt and which are labeled by their time of arrival at wave packet 1. They propagate with the velocity of light to the right from 1 to 2 for $\Delta\xi^+(t)$ and to the left from 2 to 1 for $\Delta\xi^-(t)$.

In the following equations we approximate using the small velocities of the wave-packets relative to light. It follows that to a good approximation the distance

between the wave packets can be assumed constant for the time of propagation of light between them, or for a small multiple of that time.

If $x(t)$ is the distance between the wave packets at time t (in time units) and t_j is the time that a + fluctuation passes j , then in this approximation the time of propagation can be taken as $x(t)$, where t is t_1 or t_2 or any time in between. Similarly in the propagation of a + fluctuation from 1 to 2, followed immediately by the reverse propagation of a - fluctuation from 2 to 1, takes time $2x(t)$, where t is any time during the propagation of either fluctuation between the wave-packets. In the following we use the definitions

$$t^+ = t + x(t), \quad t^- = t - x(t), \quad t^{++} = t + 2x(t), \quad t^{--} = t - 2x(t). \quad (7.3)$$

It follows that in this approximation

$$t^+ < t' \Rightarrow t < t'^-, \quad t^+ < t'^- \Rightarrow t < t'^{--}, \quad (\text{etc.}) \quad (7.4)$$

A characteristic function $\chi(\text{condition})$ is 1 when the condition is satisfied and 0 when it is not. The above approximation will be used in the characteristic functions of eqns (7.9) and (7.10) for the second order expansion with noncommuting fluctuations.

For each time t there are two fluctuations $\Delta\xi^+(t)$ and $\Delta\xi^-(t)$ that propagate from 1 to 2 and from 2 to 1 respectively with the velocity of light. The Hamiltonian is zero, each fluctuation is applied just twice, once on each wave packet. The evolution of this model system is represented by a differential fluctuation operator

$$\Delta F(t) = \Gamma^{\frac{1}{2}} [P_1(\Delta\xi^+(t) + \Delta\xi^-(t)) + P_2(\Delta\xi^+(t^-) + \Delta\xi^-(t^+))], \quad (7.5)$$

The fluctuations $\Delta\xi^\pm$ may or may not commute, but even when they do not commute with each other, they commute with the projection operators and the density operator, so they live in an 'iso-space' that is distinct from ordinary position space.

When the fluctuations commute it is convenient to define a total fluctuation operator

at time T' given by

$$\begin{aligned}
\Gamma^{-1/2} F(T') &= \Gamma^{-1/2} \lim \sum_0^{T'} \Delta F(t) \\
&= \lim \sum_0^{T'} [P_1(\Delta\xi^+(t) + \Delta\xi^-(t)) + P_2(\Delta\xi^+(t) + \Delta\xi^-(t))] \\
&= I \lim \sum_0^{T'} (\Delta\xi^+(t) + \Delta\xi^-(t)) \\
&= I \int_0^{T'} (\Delta\xi^+(t) + \Delta\xi^-(t))
\end{aligned} \tag{7.6}$$

where I is the unit operator.

For commuting fluctuations we can take the exponential without time ordering to get the propagator K_F , which is

$$K_F(T', 0) = \exp F(T') = I \exp \Gamma^{1/2} \int_0^{T'} (\Delta\xi^+(t) + \Delta\xi^-(t)), \tag{7.7}$$

and proportional to the unit operator, so by Section 5 the full propagator is unaffected by the fluctuations. So the density operator remains at ρ_0 in the interaction representation, and the interference pattern is unchanged when $T' = T$.

Now consider the perturbation expansion (4.10) of ρ_T for non-commuting fluctuations. The commutation properties do not affect the first two terms in the expansion, which therefore make zero change in the density operator.

The first significant term in the expansion is therefore $\rho_{FT}^{(2)}$, which is obtained using (6.10), and the definitions

$$\rho_{\text{dg}} = P_1 \rho_0 P_1 + P_2 \rho_0 P_2 \quad \rho_{\text{od}} = P_1 \rho_0 P_2 + P_2 \rho_0 P_1. \tag{7.8}$$

It is

$$\begin{aligned}
\Gamma^{-2}\rho_{FT}^{(2)} &= \Gamma^{-2} \lim \sum_{rr'tt'} M \chi(r < r') \delta F(r') \delta F(r) \rho_0 \chi(t < t') \delta F^\dagger(t) \delta F^\dagger(t') \\
&= \lim \sum_{rr'tt'} (\Delta t)^2 \chi(r < r') \chi(t < t') \left[4\rho_{\text{dg}}(\delta_{rt}\delta_{r't'} + \eta\delta_{rt'}\delta_{r't}) \right. \\
&\quad \left. + \rho_{\text{od}}[(\delta_{r,t-} + \delta_{r-,t})(\delta_{r',t'-} + \delta_{r'-,t'}) \right. \\
&\quad \left. + \eta(\delta_{r',t-} + \delta_{r'-,t})(\delta_{r,t'-} + \delta_{r-,t'})] \right] \\
&= \lim \sum_{tt'} (\Delta t)^2 \chi(t < t') \left[4\rho_{\text{dg}}[1 + 0] \right. \\
&\quad \left. + \rho_{\text{od}}[\chi(t^+ < t'^+) + \chi(t^+ < t'^-) + \chi(t^- < t'^+) + \chi(t^- < t'^-) \right. \\
&\quad \left. + \eta(\chi(t'^+ < t^+) + \chi(t'^+ < t^-) + \chi(t'^- < t^+) + \chi(t'^- < t^-))] \right] \\
&= \lim \sum_{tt'} (\Delta t)^2 \chi(t < t') \left[4\rho_{\text{dg}} \right. \\
&\quad \left. + \rho_{\text{od}}[\chi(t < t') + \chi(t < t'^{-}) + \chi(t < t'^{++}) + \chi(t < t') \right. \\
&\quad \left. + \eta(\chi(t' < t) + \chi(t'^{++} < t) + \chi(t'^{-} < t) + \chi(t' < t))] \right]. \tag{7.9}
\end{aligned}$$

The product of characteristic functions with incompatible arguments is zero, and the product of a characteristic function with a stronger condition and a characteristic function with a weaker condition is equal to the stronger, so

$$\begin{aligned}
\Gamma^{-2}\rho_{FT}^{(2)} &= \lim \sum_{tt'} (\Delta t)^2 \left[4\rho_{\text{dg}}[\chi(t < t')] \right. \\
&\quad \left. + \rho_{\text{od}}[3\chi(t < t') + \chi(t < t'^{-}) + \eta\chi(t'^{-} < t < t')] \right] \\
&= \lim \sum_{tt'} (\Delta t)^2 \left[4\rho_0\chi(t < t') + \rho_{\text{od}}(\eta - 1)\chi(t'^{-} < t < t') \right] \tag{7.10} \\
&= \int_0^T dt' \int_0^{t'} dt 4\rho_0 + \int_0^T dt' \int_{t'-2x(t')}^{t'} dt \rho_{\text{od}}(\eta - 1) \\
&= 2T^2\rho_0 + 2\mathcal{A}(\eta - 1)(P_1\rho_0P_2 + P_2\rho_0P_1),
\end{aligned}$$

where \mathcal{A} is the area enclosed by the two paths of the interferometer in space-time, measured in units of time². The first term is proportional to the initial density operator, and so is canceled by the corresponding term in S_R , as shown in Section

5. The second term does not affect $\text{Tr}\rho_T$, and so does not contribute to S_R . So we have

$$\begin{aligned}\rho_T &\approx \rho_0 + \Gamma^2 \mathcal{A}(\eta - 1)(P_1 \rho_0 P_2 + P_2 \rho_0 P_1) \\ \rho_T &\approx \rho_0 - \Gamma^2 \mathcal{A} \frac{8}{3}(P_1 \rho_0 P_2 + P_2 \rho_0 P_1) \quad (\text{Pauli}).\end{aligned}\tag{7.11}$$

The state diffusion due to the fluctuations tends to suppress the off-diagonal elements of the density operator, and thus decoheres the wave-packets in the two arms of the interferometer. This will be detectable by a suppression of the interference pattern if $\Gamma^2 \mathcal{A}(1 - \eta)$ is sufficiently large. Any value comparable to 1 would be enough.

8. Numerical values and discussion

At present there is no known positive evidence from experiment that space-time fluctuations of any kind suppress the interference of matter interferometers. The negative evidence, that any suppression is below the experimental limits, puts a very provisional upper bound on the possible value of the fundamental time constant τ_0 for the space-time fluctuations. The bound is provisional because it depends on many assumptions. It is assumed that the two-dimensional model adequately represents a real interferometer.

Of the three examples in the introduction, the commuting fluctuations that propagate with the velocity of light produce no decoherence when time dilation is neglected. For the other two, suppose that the experiments put an upper bound of 10% on the reduction of the off-diagonal elements of the density operator ρ by the space-time fluctuations of eqn (7.11).

Under these assumptions

$$\begin{aligned}\Gamma T &< 0.1 && (\text{delta-fluctuations}) \\ \Gamma^2 8\mathcal{A}/3 &< 0.1 && (\text{noncommuting fluctuations})\end{aligned}\tag{8.1}$$

and so from the value of the decoherence rate Γ given by (7.2), the time constant of the space-time fluctuations is bounded by

$$\begin{aligned}\tau_0 &< \frac{0.1}{T} \left(\frac{\hbar}{Auc^2} \right)^2 \approx \frac{10^{-49} \text{s}^2}{2A^2 T} && (\text{delta-fluctuations}) \\ \tau_0 &< \left(\frac{0.3}{8\mathcal{A}} \right)^{1/2} \left(\frac{\hbar}{Auc^2} \right)^2 \approx \frac{10^{-49} \text{s}^2}{A^2 \mathcal{A}^{1/2}} && (\text{noncommuting fluctuations}),\end{aligned}\tag{8.2}$$

where A is the atomic number and u is the atomic mass unit.

The best upper bound is proportional to the inverse square of the atomic number A , as given in PSD1 and PSD2. But the dependence on area and time is different from

the formulae in those papers, which is why our conclusions are different. Another new feature is that according to the theory presented here, in which the time dilation for motion of the atoms is neglected, the propagating fluctuations must not commute if there is to be any effective bound.

The best bounds are the smallest. They are given by the more massive particles, which suggests atom interferometry rather than neutron interferometry, and a relatively large value of the area \mathcal{A} . These conditions were met by the experiment of Kasevich and Chu (1992), for which the atom was sodium with $A = 23$ and the area was approximately 10^{-12}s^2 and the time $T \approx 50\text{ms}$ so that

$$\begin{aligned}\tau_0 &< 1.8 \times 10^{-51}\text{s} && (\text{delta-fluctuations}) \\ \tau_0 &< 1.8 \times 10^{-46}\text{s} && (\text{noncommuting fluctuations}).\end{aligned}\tag{8.3}$$

These are significantly less than the Planck time of about $5 \times 10^{-44}\text{s}$, which shows that this experiment puts a severe bound on possible space-time fluctuations, given the assumptions.

What about the assumptions? Theories of space-time with non-commuting metrics or an equivalent include superstring theory, GUT and the theory of Connes. They are inspired by the need to unite quantum theory with gravity. The metrics chosen for this paper were obtained by the conditions of simplicity and accessibility by measurement, and from a perceived need for a realistic quantum theory in the sense of Einstein, Podolsky and Rosen (1935) and of Bell (1987). Any similarity to other non-commuting metrics is a bonus.

The main weakness of this paper is the crudity of the model matter interferometer, because we have taken the Hamiltonian to be the mass together with projectors onto the wave packets in the arms of the interferometer, neglecting the kinetic energy terms. This is a non-trivial assumption.

However, despite these assumptions, we have shown by means of the model that Einstein's method of accessing small quantities using diffusion processes, which he applied so successfully to the atomic scale using Brownian motion, might also be applied today to carry out experiments on Planck scale space-time fluctuations. The small value of the Planck time is not an impassable barrier to the precision of modern matter interferometers. Even if the experiments were to show that such fluctuations are not significant on a Planck scale and below, which looks possible, this would be a contribution to a field that has lacked such experimental evidence in the past.

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